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A Mathematical Description for Taste Perception Using Stochastic Leaky Integrate-and-Fire Model

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Maryeme Ouafoudi [*†]

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Abstract

How the brain makes sense of a complicated environment is an important research question to us, and a first step is to be able to reconstructing the stimulus that give rise to an observed brain response. Neural coding relates neurobiological observations to external stimuli using computational methods. Encoding refers to how a stimulus affects the neuronal output, and entails constructing a neural model and parameters estimation. Decoding refers to reconstruction of the stimulus that led to a given neuronal output. Here we perform neural

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encoding/decoding model for a mixture of multiple stimuli (4 types of taste: sweet, sour, salty and bitter) using leaky integrate and fire models describing neural spike trains. Our model's performance will be compared to the work of Dr. Kathrin Ohla [1].

1 Introduction

Taste is unique among sensory in its innate association with mechanisms of reward and aversion in addition to its recognition of quality; Sweet taste indicates the availability of carbohydrates, salty taste allows electrolyte detection, umami taste serves protein recognition, and sour and bitter tastes alert us to acids and potentially harmful substances like alkaloids, respectively [2]. This paper performs an opportunity to combine between neuronal modeling of Human taste and experimental data obtained under controlled conditions, thanks to the collaboration with the lab of Dr. Kathrin Ohla. The considered data ara EEG data. A progress in the modeling of human taste signaling is our first goal. Here, we focus on the last stage of taste transduction. The four taste qualities (sweet, sour, salty, bitter) contribute to the membrane voltage that we model as a stochastic process with a diffusion component. Leaky-integrate-and-fire model is our considered model for the firing mechanism. We would like to remark, that mathematical models for sensory perception exist in large to small numbers for vision, hearing

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and olfaction, respectively. Modelling taste perception appears to be not addressed in the literature.

2 Leaky Integrate-and-fire model

The leaky integrate-and-fire (LIF) models are simple diffusion models for the dynamics of the membrane potential in single neurons [3][4], the most common being Ornstein-Uhlenbeck processes (OU) with constant conductance, leak potential, and diffusion coefficient. However, the model can be extended by incorporating post-spike currents with a spike-response kernel function [5].

3 Stochastic Stimulus

A stimulus is stochastic if it involves strong noise besides its deterministic part, such a stimulus is described by a stochastic diffusion process. Decoding a stochastic stimulus requires estimating its parameters and recovering the realization of the stocchastic stumuli at each time step. Stimuli differ from each others, a stimulus may define a sound, position of object, smell of perfume. In our case, a stimulus is defined by a tastant (sweet, sour, bitter or salty). Those signals are more realistically represented by a stochastic process than a deterministic functions. In this work we consider mixtures of stochastic stimuli represented by an OU process evolving continuously over time.

4 Neural Decoding

Decoding refers to the problem of how to read out the information contained in a set of spike trains and has both theoretical implications for the study of neural decoding. The neural decoding plays an important role in understanding the mechanisms of neurons and the brain. Well-performing algorithms of decoding involve necessary components of brain-machine interfaces [6][7] . Different methods have been used in this area of computational neuroscience. Some methods focus on regression-related approaches building linear models between spike trains and the corresponding stimulus by optimal linear estimation(OLE) [8][9]. Machine learning method are also used for decoding stimulli, i.e., artificial neural networks [10], kernel regression, and kernel-based neural metrics [11][12]. These methods use general statistical techniques and exclude the specefic spike-generating mechanism of the neural response. Contrarily, stimulus decoding employs spiking neural models that describe the spike generating mechanisms from stimuli [13][14]. Different encoding models can be used. Approximate methods using point processes treat the spikes in a spike train as sequential random events, which can be equivalently formulated as generalized linear models (GLM) for model fitting [14][15]. In the meantime, there exists othe neuorobiological methods as integrate-and-fire models which study the evolution of the membrane potential. In decoding methods, the encoding models are used in the posterior distribution to infer the most likely stimuli. Decoding of constant stimulus can be obtained from the posterior distribution by using the Maximum A Posteriori (MAP) or Sequential Monte Carlo Methods. The decoding of temporal stimuli can be discretized as a sequence of constant decoding tasks, which can be solved by Kalman filtering [16] or particle Monte Carlo methods [17][18][19][20]. Previous research at the Institute of Stochastics has considered Kalman filtering and Apprximate Bayesian Computation to estimate the state and parameters of stochastic differential equations modelling neuronal models. In each case the part of the contribution involved developing stochastic numerical methods to perform the statistical procedures efficiently.

5 Gustatory Cortex Model

We constructed a GC model by means of attractor dynamics that has been shown to play effective roles in memory storage and sensory processing [21] [22]. Assuming that the GC network has 2-dimensional array of GC neurons, it contains M neurons in each row and N in each column. GC neurons receive the output of the thalamus, as represented in the figure 2, and represent the gustatory information of taste quality and intensity. The GC model has two main simplified features for the anatomical structures of GC. Firstly, the GC network involves neural units, each has a pair of main neuron (excitatory) and interneuron (inhibitory) as it is shown in the figure 3. The main neuron provides an excitatory output for an interneuron, and receives an inhibitory input. According to [23], the interneurons are distributed in a random manner in the GC and they provide the neighbouring neurons with inhibition. In the following presented model, the local inhibition from the neihboring neurons was modeled with an inhibitory paired with GC neurons. Secondly is on the connection between the GC neurons. It's not yet understood how the GC neurons are connected to each other in the gustatory cortex. Even though, we assumed that the main neurons are fully connected with excitatory synapses, figure 3. In our model, we assume that the connection of GC neurons is shaped by experience dependent learning. Hence, the only GC neurons responsive to the same taste quality are strongly connected to each other after the learning.

The way how the input or external signal determines the membrane voltage of the receptor neuron (i, j), V_{ij}^{GC}, is modeled by a stochastic leaky integrate and fire (LIF) model

$$dV_{ij}^{GC} = \left[-\frac{1}{\tau_{ij}^{GC}} \left(V_{ij}^{GC} - V_0 \right) + F_{ij} + I(t) + H(t) \right] dt + \sigma dW$$

$$F_{ij} = \sum_{t=t_1}^{t_m} \sum_{kl} \omega_{ij,kl}(t, t_d) f(V_{kl}(t - t_d))$$

$$\frac{dV_{ij}^{IN}}{dt} = -\frac{1}{\tau_{IN}} \left(V_{ij}^{IN} - V_0 \right) + \omega_E f(V_{ij}^{GC}(t))$$

Where $\omega_{ij,kl}(t, t_d)$ is the weight of the synaptic connection from the (k, l)th GC neuron to the (i, j)th one, with the time delay t_d. The time delay has a multiple delays described by $t_1, t_2, \ldots, t_m$, where t_i $(i = 1 - m;\ m = 10)$ take the values of $0 - 9$ ms with the time interval of 1 ms. The synaptic weight ω_E is the weight of the excitatory connection from main neuron to the paired interneuron. τ_{GC} and τ_{IN} are the time constants of the membrane potentials , V_{ij}^{GC} and V_{ij}^{IN}, respectively. The stimulus current $I(t)$ is shaped from the external stimulus $S(t)$ through a stimulus kernel $k_s(t)$; $I(t) = \int_{-\infty}^{t} k_s(t)(t-s)S(s)ds$. The post-spike current arises from past spikes convoluted with a response kernel $k_h(t)$; $H(t) = \int_{-\infty}^{t} k_h(t-s)\mathrm{I}(s)ds$. Here

$\mathrm{I}(s) = \sum_{\tau \in \{t_1, t_2, \ldots\}} \delta(s-\tau)$ represents the spike train, where $\delta(.)$ denotes the Dirac delta function. Here, $W = \{W(t), t \geq 0\}$ is a one-dimensional Wiener process, while the drift componnet and the diffusion coefficient fullfill the necessary global Lipschitz and linear growth conditions to ensure the existence of the unique solution [24]. Assuming that the stimulus kernel is without delay, such that $k_s(t) = \delta(t)$, which implies that $I(t) = S(t)$. The response kernel is considered to be the difference of two exponentials decaying over time,

$$k_h(t) = \eta_1 e^{-\eta_2 t} - \eta_3 e^{-\eta_4} t$$

with four positive parameters, $\eta = (\eta_1, \eta_2, \eta_3, \eta_4)$. By adjusting the parameters, different kernels are obtained.

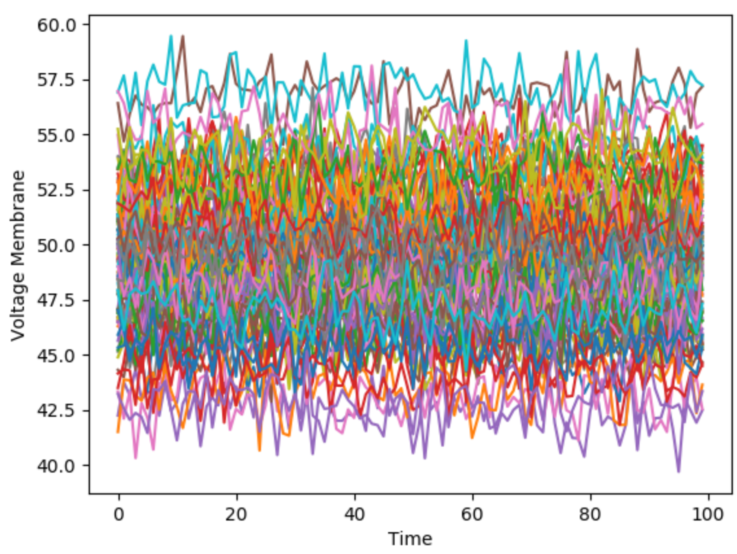

Figure 1: Realization of the voltage membrane at the GC.

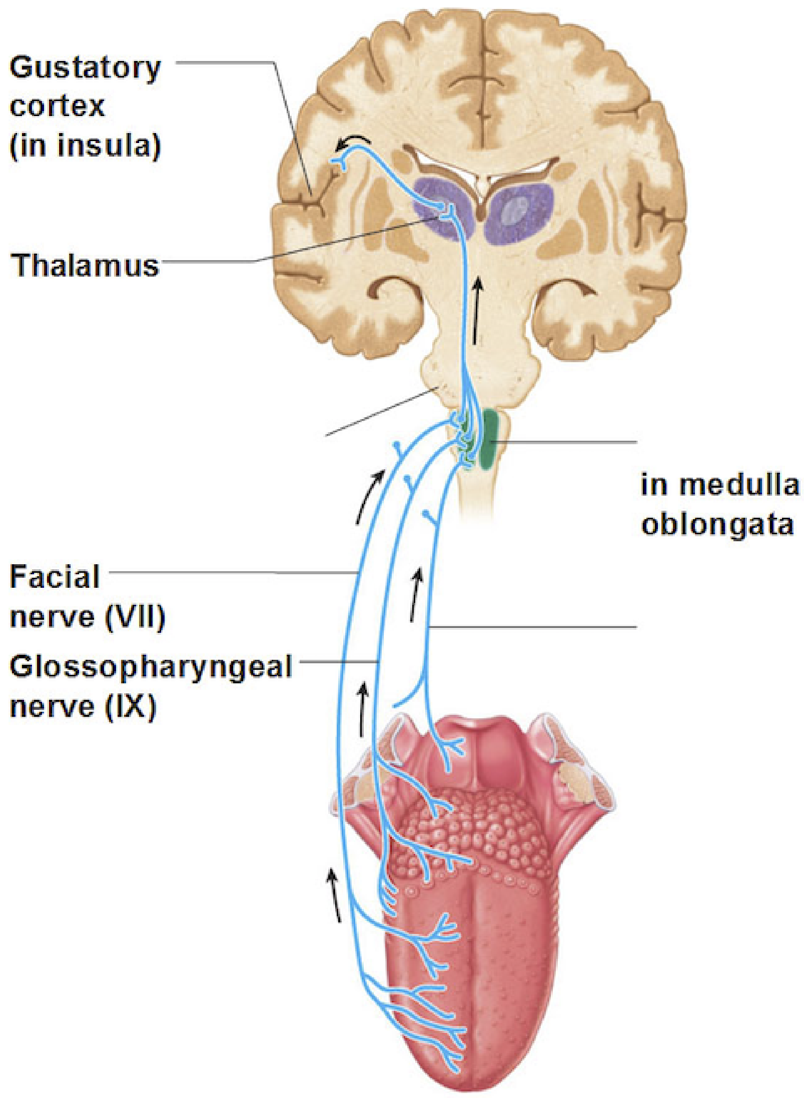

Figure 2: Taste Pathways

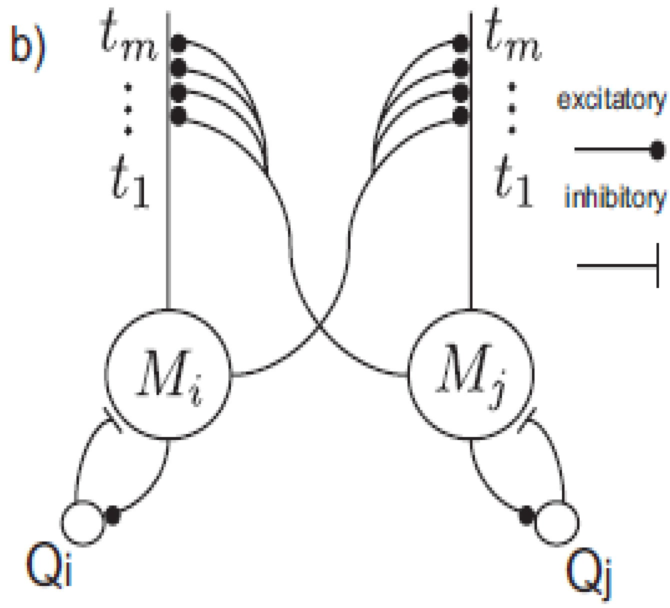

Figure 3: (b) The neural units of the GC model and their connections. The ith unit consists of a main neuron, M_i, and an inhibitory neuron Q_i. The main neuron M_i provides the interneuron Q_i with an excitation, and the interneuron Q_i gives the neuron M_i an inhibition. The main neurons, M_i and M_j, are connected with the synapses with different time delays, $t_1, \ldots, t_m$.

6 Likelihood of an Observed Spike Train

We suppose that there are a total of K stimuli inside a neuron, and denote by $S = \left(S^1, S^2, \ldots, S^K\right)$. Define, Y as $\left(Y^1, Y^2, \ldots, Y^M\right)$, the realization of the stimuli and spike trains are respectively $s = \left(s^1, s^2, \ldots, s^K\right)$ and $y = \left(y^1, y^2, \ldots, y^M\right)$. Following the probability-mixing encoding model. the stimuls driven current, $I(t)$, follows a probability mixture:

$$I(t) = S^k(t), \text{with probability } \alpha_k,$$

where $\sum_{k=1}^{K} \alpha_k = 1$. So, the probability of each spike train is also a mixture distribution given by,

$$p(y^m|s) = \sum_{k=1}^{K} \alpha_k p(y^m|s^k),$$

where $p(y^m|s^k)$, represents the probability of generating spike train y^m from the single stimulus s_k, defined as the product of the probability densities of all spike times $y^m = (t_1, t_2, \dots)$. However, the dependance between spike times is accounted for by conditioning on the history of the the past spike times, denoted by $H_{t_i-1,m}$,

$$p(y^m|s^k) = \prod_i f(t_i|s^k, H_{t_i-1,m}).$$

$f(t|s^k, H_{t_i-1})$ represents the conditional probability density of spiking at a given time t within the $k-$th stimulus and the past spike up to the spike time t_{i-1}. In case of the independance of the spike trains $Y = \left(Y^1, Y^2, \dots, Y^M\right)$, the likelihood for the realizations $y = (y^1, y^2, \dots, y^m)$ will be given by

$$p(y|s) = \prod_{m=1}^{M} p(y^m|s) = \prod_{m=1}^{M} \sum_{k=1}^{K} \alpha_k p(y^m|s^k),$$

Decoding of the stochastic stimulus

Considering a stochastic stimulus mixtures, described by stochastic processes with unknown paramaters. Our aim is both estimating the parameters governing the law of the $k-$th stimulus as well as decoding its stochastic realization. The stochastic stimuli are described by Ornstein-Uhlenbeck processes, for a mixture of K stimuli $S = \left(S^1, S^2, \ldots, S^K\right)$, the $k-$th stimulus component is governed by the folllowing stochastic differential equation,

$$dS^K = \left(\mu^k - S^k(t)\right) dt + \gamma dW(t)$$

μ^k and γ represent parameters of the stimuli, and $W(t)$ is a standard Wiener process. Note that the drift parameter is a stumlus specific and the diffusion parameter is considered to be the same for all the stimuli in the mixture.

The parameters describing the stimulus are unknown, namely $\theta = (\gamma, \mu) \in \mathrm{R} \times \mathrm{R}^K$. For decoding the stumuli, we focus on different statistical methods, to name **filtering and smoothing theories** for more details see [25].

6.1 State space model

State space model are class of probabilistic models that describe the probabilistic dependence between the latent state variable and the observed measurement. We use this type of model to describe the dynamical evolution of the stochastic stimuli. The state space is aimed to include the unknown stimulus related parameters that are included for the construction of the decoding algorithm, besides the stimulus. The stimuli S are continuous states Markov processes. The transition of the stimuli states is parametrized by $\theta = (\gamma, \mu)$. Denote by $Z_n = (S_n, \theta_n)$ the full hidden states, and by z_n its realization. The full states are:

γ_n	common diffusion parameter of all stimuli
$\beta_n = \left(\beta_n^1, \beta_n^2, \ldots, \beta_n^K\right)$	drift parameter of all stimuli

$$S_n = \left(S_n^1, S_n^2, \ldots, S_n^K\right) \qquad \text{value of each stimulus}$$

The index n refers to the current time in the state evolution. Note that the propagation of the states at time n is given by:

$$\begin{aligned}
\gamma_n &\sim N_{tr}\left(\gamma_{n-1}, V_\gamma\right) \\
\beta_n^k &\sim N\left(\beta_{n-1}, V_\beta\right) \\
S_n^k &\sim N\left(M_n^k, V_n^k\right)
\end{aligned}$$

for $k \in \{1, \ldots, K\}$. The parameters γ_n and β_n follow a Gaussian distribution with variances V_γ and V_β. The strength of each stimulus, is updated according to the OU model following a Gaussian distribution with the following mean and variance,

$$\begin{aligned}
M_n^k &= \left(S_{n-1}^k - \beta_n^k\right) e^{-\delta t} + \beta_n^k \\
V_n^k &= \gamma_n^2 \frac{\left(1 - e^{-2\delta t}\right)}{2}.
\end{aligned}$$

Given the parameters, the likelihood of the spike train is obtained via the encoding model. Based on the small discretization of the time interval, we have to take into account the boundary effects, meaning the time from the left boundary of the interval to the first spike, and the time from last spike to the right boundary. Denote by T_b and T_e the begining and the end of the interval, respectively. If y_n is not empty, i.e. $T_b \leq t_1 < t_2 < \cdots < t_{L_n} \leq T_e$. Given a stimulus $S_{1:n} = s_{1:n}$, the likelihood of y_n is given as follows

$$
\begin{aligned}
p(y_n|s_n, s_{n-1}, H_{T_b}) &= \prod_{l=2}^{L_n} g(t_l|s_n, H_{t_{l-1}}) && \text{(Complete ISI)} \\
&\times \; g(t_1|s_n, s_{n-1}, H_{T_b}) && \text{(Left Boundary)} \\
&\times \; \left[1 - \int_{t_{L_n}}^{T_e} g(\tau|s_n, H_{t_{L_n}}) d\tau\right] && \text{(SP, right boundary)},
\end{aligned}
$$

whre SP means the survival probability. In the absence of spikes, the likelhood is given by the survival probability

$$
p(s_{1:n}|y_{1:n}) = \int_\Omega p(z_{1:n}|y_{1:n}) d\theta = \int_\Omega p(s_{1:n}|y_{1:n}, \theta_{1:n}) p(\theta_{1:n}|y_{1:n}) d\theta,
$$

Where $\Omega = \mathrm{R} \times \mathrm{R}^K$ is the parameter space.

7 Conclusion

Our goal focuses on setting up a forward backward mathematical model, encoding decoding model, in order to obatin output similar to the experimental results achieved by Dr. Kathrin Ohla. Both problems require a deep synthesis of the state-of-art mathematical and neuroscientific expertise.

References

[1] S. M. Crouzet, N. A. Busch, and K. Ohla, "Taste quality decoding parallels taste sensations," *Current Biology*, vol. 25, no. 7, pp. 890–896, 2015.

[2] P. A. Breslin, “An evolutionary perspective on food and human taste,” *Current Biology*, vol. 23, no. 9, pp. R409–R418, 2013.

[3] A. N. Burkitt, “A review of the integrate-and-fire neuron model: I. homogeneous synaptic input,” *Biological cybernetics*, vol. 95, no. 1, pp. 1–19, 2006.

[4] L. Sacerdote and M. T. Giraudo, “Stochastic integrate and fire models: a review on mathematical methods and their applications,” in *Stochastic biomathematical models*. Springer, 2013, pp. 99–148.

[5] W. M. Kistler, W. Gerstner, and J. L. v. Hemmen, “Reduction of the hodgkin-huxley equations to a single-variable threshold model,” *Neural computation*, vol. 9, no. 5, pp. 1015–1045, 1997.

[6] M. A. Lebedev and M. A. Nicolelis, “Brain–machine interfaces: past, present and future,” *TRENDS in Neurosciences*, vol. 29, no. 9, pp. 536–546, 2006.

[7] S. Waldert, T. Pistohl, C. Braun, T. Ball, A. Aertsen, and C. Mehring, “A review on directional information in neural signals for brain-machine interfaces,” *Journal of Physiology-Paris*, vol. 103, no. 3-5, pp. 244–254, 2009.

[8] A. P. Georgopoulos, A. B. Schwartz, and R. E. Kettner, “Neuronal population coding of movement direction,” *Science*, vol. 233, no. 4771, pp. 1416–1419, 1986.

[9] F. Rieke, D. Warland, R. D. R. Van Steveninck, W. S. Bialek *et al.*, *Spikes: exploring the neural code.* MIT press Cambridge, 1999, vol. 7, no. 1.

[10] D. K. Warland, P. Reinagel, and M. Meister, "Decoding visual information from a population of retinal ganglion cells," *Journal of neurophysiology*, vol. 78, no. 5, pp. 2336–2350, 1997.

[11] J. Eichhorn, A. Tolias, A. Zien, M. Kuss, J. Weston, N. Logothetis, B. Schölkopf, and C. E. Rasmussen, "Prediction on spike data using kernel algorithms," in *Advances in neural information processing systems*, 2004, pp. 1367–1374.

[12] A. J. Brockmeier, J. S. Choi, E. G. Kriminger, J. T. Francis, and J. C. Principe, "Neural decoding with kernel-based metric learning," *Neural computation*, vol. 26, no. 6, pp. 1080–1107, 2014.

[13] S. Koyama, U. T. Eden, E. N. Brown, and R. E. Kass, "Bayesian decoding of neural spike trains," *Annals of the Institute of Statistical Mathematics*, vol. 62, no. 1, p. 37, 2010.

[14] W. Truccolo, U. T. Eden, M. R. Fellows, J. P. Donoghue, and E. N. Brown, "A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects," *Journal of neurophysiology*, vol. 93, no. 2, pp. 1074–1089, 2005.

[15] R. E. Kass, U. T. Eden, and E. N. Brown, *Analysis of neural data.* Springer, 2014, vol. 491.

[16] W. Wu, Y. Gao, E. Bienenstock, J. P. Donoghue, and M. J. Black, "Bayesian population decoding of motor cortical activity using a kalman filter," *Neural computation*, vol. 18, no. 1, pp. 80–118, 2006.

[17] L. Paninski, Y. Ahmadian, D. G. Ferreira, S. Koyama, K. R. Rad, M. Vidne, J. Vogelstein, and W. Wu, "A new look at state-space models for neural data," *Journal of computational neuroscience*, vol. 29, no. 1-2, pp. 107–126, 2010.

[18] R. C. Kelly and T. S. Lee, "Decoding v1 neuronal activity using particle filtering with volterra kernels," in *Advances in neural information processing systems*, 2004, pp. 1359–1366.

[19] A. E. Brockwell, A. L. Rojas, and R. E. Kass, "Recursive bayesian decoding of motor cortical signals by particle filtering," *Journal of Neurophysiology*, vol. 91, no. 4, pp. 1899–1907, 2004.

[20] S. Shoham, L. M. Paninski, M. R. Fellows, N. G. Hatsopoulos, J. P. Donoghue, and R. A. Normann, "Statistical encoding model for a primary motor cortical brain-machine interface," *IEEE Transactions on Biomedical Engineering*, vol. 52, no. 7, pp. 1312–1322, 2005.

[21] O. Hoshino, Y. Kashimori, and T. Kambara, "Self-organized phase transitions in neural networks as a neural mechanism of information processing," *Proceedings of the National Academy of Sciences*, vol. 93, no. 8, pp. 3303–3307, 1996.

[22] ——, “An olfactory recognition model based on spatio-temporal encoding of odor quality in the olfactory bulb,” *Biological Cybernetics*, vol. 79, no. 2, pp. 109–120, 1998.

[23] H. Markram, M. Toledo-Rodriguez, Y. Wang, A. Gupta, G. Silberberg, and C. Wu, “Interneurons of the neocortical inhibitory system,” *Nature reviews neuroscience*, vol. 5, no. 10, pp. 793–807, 2004.

[24] L. Arnold, “Stochastic differential equations: theory and applications. 1974,” *A Wiley-Interascience Publications*.

[25] S. Särkkä, *Bayesian filtering and smoothing.* Cambridge University Press, 2013, vol. 3.

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