Basics

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Education

- Beihang University (Bachelor's degree study)
- Trinity College Dublin, on exchange (one semester, Bachelor's thesis)
- Bachelor's thesis: UTP semantics of non-deterministic side-effecting expression
- Key Laboratory of Mathematics-Mechanization, Chinese Academy of Sciences (Master's degree study)
- Master thesis: Group-theoretical Method for Matrix Multiplication
- PhD student in DK9

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Main results

- An example leading to a non-trivial bound: $\omega \leq 2.9262$
- TPP and DPP property of Sylow subgroups of a given group.
- 6×6 small matrix multiplication: Reduces to 56 candidates for groups of order < 90.
- Relations between the TPP of an abstract group B and the group $C_n \times B$.

Main results

- An example leading to a non-trivial bound: $\omega \leq 2.9262$
- TPP and DPP property of Sylow subgroups of a given group.
- 6×6 small matrix multiplication: Reduces to 56 candidates for groups of order < 90.
- Relations between the TPP(Triple Product Property) of an abstract group B and the group $C_n \times B$.

Triple Product Property

- If S is a subset of a group, let $Q(S)$ denote the right quotient set of S,i.e., $Q(S) = \{s_1 s_2^{-1} : s_1, s_2 \in S\}$.

Definition (CU03, Definition 2.1.)

A group realizes $\langle n_1, n_2, n_3 \rangle$ if there are subsets $S_1, S_2, S_3 \subseteq G$ such that $|S_i| = n_i$, and for $q_i \in Q(S_i)$, if $q_1 q_2 q_3 = 1$ then $q_1 = q_2 = q_3 = 1$. We call this condition on S_1, S_2, S_3 the **triple product property**.

Constructing TPP

Theorem

A_4(alternating group of order 4) realizes $\langle 3, 3, 2 \rangle$.

Proof.

TPP triples: $S : \{(1), (13)(24)\}$; $T : \{(1), (243), (234)\}$; $U : \{(1), (124), (142)\}$. □

Contructing TPP

Denote $G := C_6 \times A_4$.

Proposition

G realizes $\langle 6, 6, 3\rangle$ via S_1, T_1, U_1:
$S_1 :=$
$\{(1,1),(1,(13)(24)),(\bar{3}^{(1)},1),(\bar{3}^{(1)},(13)(24)),(\bar{3}^{(2)},1),(\bar{3}^{(2)},(13)(24))\}$;
$T_1 :=$
$\{(1,1),(1,(243)),(1,(234)),(\bar{2}^{(1)},1),(\bar{2}^{(1)},(243)),(\bar{2}^{(1)},(234))\}$;
$U_1 := \{(1,1),(1,(124)),(1,(142))\}$.

Constructing TPP triples

Denote $H := C_3 \times A_4$.

Proposition

H realizes $\langle 6, 4, 3 \rangle$ via S, T, U:

$S :=$
$\{(1,1), (1,(13)(24)), (\bar{3}^{(1)}, (13)(24)), (\bar{3}^{(2)}, (13)(24)), (\bar{3}^{(1)}, 1), (\bar{3}^{(2)}, 1)\}$;
$T := \{(1,1), (1,(14)(23)), (1,(143)), (1,(134))\}$;
$U := \{(1,1), (1,(123)), (1,(132))\}$.

Motivation

- From the examples above we can see that once I got a "TPP" triple of a subgoup, say A_4 , I would like to expand it in some way to get a "TPP" triple of a bigger group, say $C_6 \times A_4$ or $C_3 \times A_4$.
- It's easier to obtain a TPP triple of a smaller group, so I would like to find some theory behind, say relations between TPP of A_4 and TPP of $C_n \times A_4$. (C_n: cyclic group of order n)

constructing TPP triples—some principles

$D := C_2 \times B,\ Q := C_3 \times B,\ F := C_n \times B.$
Take $\langle 6, 6, 6 \rangle$ for S_2, T_2, U_2 for example:

S_2	T_2	U_2
$(1, s_1)$	$(1, t_1)$	$(1, u_1)$
$(1, s_2)$	$(1, t_2)$	$(1, u_2)$
$(1, s_3)$	$(1, t_3)$	$(2, z_1)$
$(2, x_1)$	$(2, y_1)$	$(2, z_2)$
$(2, x_2)$	$(2, y_2)$	$(2, z_3)$
$(2, x_3)$	$(2, y_3)$	$(2, z_4)$

Here,we have $S = \{s_1, s_2, s_3\}, T = \{t_1, t_2, t_3\}, U = \{u_1, u_2\}, X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}, Z = \{z_1, z_2, z_3, z_4\}$. And $C_2 = \{1, 2\}$ is the cyclic group of order 2, 1 is the unit and 2 represents the 2-ordered element in it.

constructing TPP triples—some principles

Theorem

If $S_2, T_2, U_2 \subset D$ satisfy TPP and $S \cap X \neq \phi$, then $Y \cap T = \phi$ and $Z \cap U = \phi$ must hold.

Proof.

When $S_2, T_2, U_2 \subset D$ has TPP property, if $S \cap X \neq \phi$. Suppose $Y \cap T \neq \phi$, w.l.o.g., $y_1 = t_1$, $s_1 = x_1$, then we have $(1, s_1)(2, x_1)^{-1}(1, t_1)(2, y_1)^{-1}(1, u_1)(1, u_1)^{-1} = 1$, but obviously $(1, s_1) \neq (2, x_1)$, contradiction! With the same approach, we can obtain $Z \cap U \neq \phi$. □

constructing TPP triples—some principles

Theorem

If $S_3, T_3, U_3 \subset Q$ satisfy TPP and $S \cap X \neq \phi$, then we have $Y \cap T = \phi$ and $Z \cap U = \phi$.

Theorem

If $S_2, T_2, U_2 \subset D$ satisfy TPP, then the subset triples (S, Y, U), (S, Y, Z), (S, T, Z), (X, T, U), (X, T, Z), (X, Y, U), (X, Y, Z) of B all satisfy TPP.

constructing TPP triples—some principles

Theorem

If $S_2, T_2, U_2 \subset D$ satisfy TPP, and $S_2|_B$ contains some repeated elements, then B realizes $\langle a, b, c\rangle$, where $a = r + 1$ (r is the number of elements that has more than one occurrence in $S_2|_B$), $b = |T_2|$, $c = |U_2|$.

An example for the theorem on the previous slide

Example

S_2	T_2	U_2
$(1, s_1)$	$(1, t_1)$	$(1, u_1)$
$(1, s_2)$	$(1, t_2)$	$(1, u_2)$
$(1, s_3)$	$(1, t_3)$	$(2, z_1)$
$(2, x_1)$	$(2, y_1)$	$(2, z_2)$
$(2, x_2)$	$(2, y_2)$	$(2, z_3)$
$(2, x_3)$	$(2, y_3)$	$(2, z_4)$

Here $|S \cap X| = r, a = r + 1, b = |T_2|, c = |U_2|$, then if $S_2, T_2, U_2 \subset D$ satisfy TPP, we can obtain that B satisfies TPP via $\langle a, b, c \rangle$, where $D = C_2 \times B$, $C_2 = \{1, 2\}$ is the cyclic group of order 2, 1 is the unit and 2 represents the 2-ordered element in it.

Reference

- (CU03) Cohn H, Umans C. A group-theoretic approach to fast matrix multiplication[C]. Foundations of Computer Science, 2003. Proceedings. 44th Annual IEEE Symposium on. IEEE, 2003: 438-449.
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Thank You