Concise Tree Models Based on L-system and Interpolation

Abstract

There are such many different shapes of the leaves. And everything in nature exists for its own reason. Qualitatively, veins play a vital role in forming the shape of a leaf. Based on fractal theory, our model elaborates most trees' profiles，and thence we gain the algorithm to solve the mass-of-leaves problem. Besides, we establish a quite flexible leaf model varying from different types of trees, which demonstrates association in profile characteristics between a certain tree and its homologous leaves. In addition, the leaf model can be seen as a criterion for classifying the leaves. Thus, we reach the two key factors for describing and classifying leaves—leaf vein's structure and leaf shape. We provide two modes for classifying leaves in total. At last we gain two functional models accounting for the correlation between the leaf mass and the size of the tree. By simulating the tree in virtual environment and calculation, we get vivid results and our models correspond well with our expectation.

Key words: Fractal theory; simulation; leaf-mass estimation; interpolation and fitting

Restatement of the problem

Everything in the nature is fabulous. Now we come to a very important organ of plants—the leaf. The basic characteristics of leaf are mass and shape. How can we study them? And leaf is of high diversity, how can we classify it? What is the relationship between the characteristics of leaf and of the tree?

Introduction

Why do leaves have the various shapes that they have? A leaf is a plant organ thin and flat located above ground for photosynthesis as well as a production organ for plant. Leaf works as area for transpiration moreover. Leaves may store food or water and a majority holds other functions apart from those outlined above. Apparently, climate and available light carve them to a degree but they have been shaped by factors such as graze of animals, ecological competition from other species and available nutrients as well. Adopted by more and more people, a new theory asserting it is leaf vein that determines leaf shape rises. John Whitfield mentioned the theory in his paper Plant Ecology: The Cost of Leafing issued on 29th November, 2009(Nature).

Scientists believe the function of a leaf is relating to basic crucial sides, namely supporting the CO_2 for leaves' growing, life span for leaves and photosynthetic rate either. Moreover, their combination through diverse patterns/dipping into dissimilar create their diversity in shapes and structures of the leaves. Benjamin Blonder set up a model in his paper Venation networks and the origin of the leaf economics spectrum claiming density, space between leaf veins and amount of mini-veins' distributing zone separately correspond with those crucial aspects mentioned before. When a plant open its stoma as to gain more CO_2 to provide for photosynthesis， it evaporates. This phenomenon calls for many pipelines being into the water. Conversely, it means the plenty need of large-sized leaves. As we know, it is essential for plants to absorbing water without a break, hence certain leaf veins form their geometry distribution and shapes of leaves are locked ultimately. Consequently, veins, skeleton of the leaf, have an absolute right to speak of a leaf's profile characteristics.

The model produced by Blonder sheds lights on our path ahead. In our view, leaves on different trees have different profiles for they hold diverse genes. As for the reason why there being discrepancies between leaves in the same tree, we consider three factors: distance from the leaf to the trunk, consistency of IAA, characteristics of the veins. However, these factors are not detailed discussed in our paper.

In classifying leaves, He Shu provided us a method in the paper Application of SOM Neural Network on Leaves' Shape Classification. He uses SOM neural network (the presently general self-organizing feature map network), by the method of characteristic extraction, and the classifying accuracy reaches 86. 67%. However, this way needs real leaves for detection. Therefore, we developed a new model basing on the fractal theory.

Fractal is a mathematical conception that holds a dimension exceeding its topological dimension and fall between the integers in most situations. In B.B.Mandelbrot's works Fractal: Form, Chance and Dimensions, its put forward by the French mathematician for the first time in history. Fractals are typical self-similar patterns and self-similar means the profile stay unchanged when it's amplified or shrank. Fractals could look exactly the same at each scale.

Typical fractals obey these rules: (Chang Jie etc. 1996):

1. It will not lose the complexity no matter what scale it's measured, that is to say, it has infinite accuracy-structure.
2. It is irregular so that we cannot describe it simply by the traditional geometry words.
3. It holds the character of self-similarity.
4. It could be produced by basic rules, namely recursion, iteration.
5. Fractal dimension is bigger than topological dimension.

Inspired by our knowledge of nature science and intuition, our tree model is developed by fractal theory. Fractal has been generally used in the research of botany these few decades, but it has shown significant effect. We try to use a popular but basic model—L-system to describe the tree. For leaf, we mainly consider the vein structure and the shape to set model. In addition, we use Matlab as the programming platform and Xfrog as the simulation platform.

Now comes to the organization of our paper. For the tree model, we start from the first generation of that fractal. Thus, we simulate our tree model step by step and we assume that leaf grow from the end of the branches. For that reason we can easily obtain the quantity of the leaves and the mass of the leaf. Then we can use this model to analyze the correlation of leaf shape and exposure area. Next we assure leaf shape be related to branching structure after using some repression statistics knowledge. Finally, we combine others research and our model and gain a results of relationship of some characteristics.

Our model

Tree model:

Assumptions:

1. A tree appears the same in profile in every perspective.
2. We assume the tree fits with the fractal model and we see the last three generations as the leaves.
3. We assume the leaf fits the fractal model either.

D0L-system:

L-systems were introduced in 1968 by an American biologist named Aristid Lindenmayer(1925–1989).It is skilled in describing plants' growth or their profiles. A L-system consists of an set of symbols that can be used to make strings, a collection of producing rules which expands each symbol into a larger string of symbols, an initial string from which to begin construction, and a mechanism for the translation of the generated strings into geometric structures. L-systems can also be used to generate self-similar fractals such as iterated function systems. D0L-system is one of the most simple graph-simulation systems.

Symbols:

symbols	meanings
$\boldsymbol{\alpha}$	We call it "heading"
$\boldsymbol{d}$	Step length
$\boldsymbol{f}$	Move forward for d step and line the path trampled.
$\boldsymbol{h}$	Move forward for d step.
$+$	Turn left by the angle of δ.
$-$	Turn right by the angle of δ.

However, our model uses 0L-system which needs some advantages towards D0L-system. We need extra symbols.

symbols	meanings
[	Push present state into the stack.
]	Pop out one state to serve as the current state.

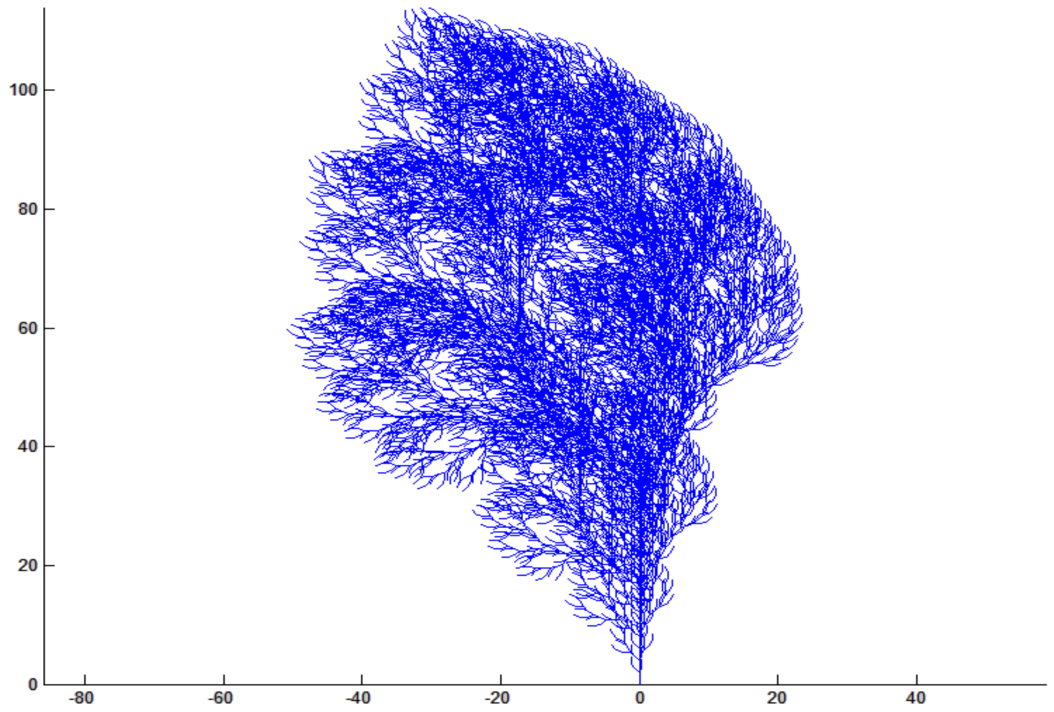
Plant simulation:

Demonstration 1($\delta = 22.5°$, *L-system*):

$$\omega = f$$

$$P: f \rightarrow ff + [+f - f - f] - [-f + f + f]$$

After iteration, a vivid image of tree appears:



Graph 1: Demonstration 1($\delta = 22.5°$, *L-system*, $\omega = f, P: f \rightarrow ff + [+f - f - f] - [-f + f + f], n = 8$)

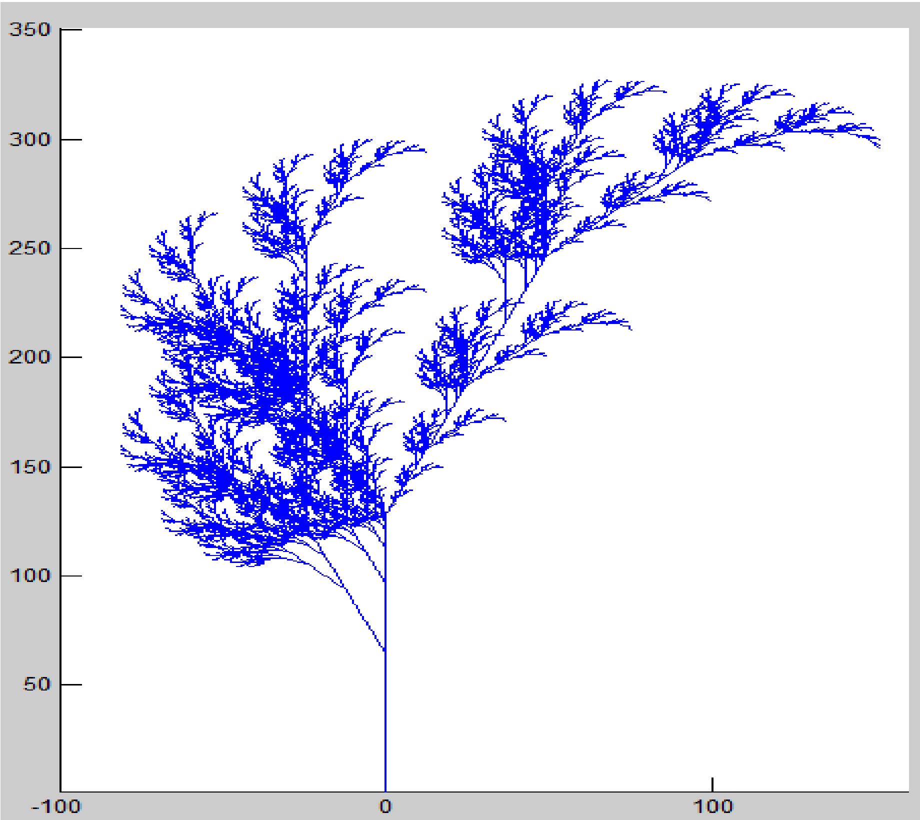
Demonstration 2($\delta = 22.5°$, *L-system*):

$$\omega = f$$

$$P_1: f \rightarrow h + [[f] - f] - h[-hf] + f$$

$$P_2: h \rightarrow hh$$

We make an iteration and the result is showed below.



Graph 2: L-system tree simulation($\delta = 22.5°,\ \omega = f, P_1: f \rightarrow h + [[f] - f] - h[-hf] + f, P_2: h \rightarrow hh$*, n=8)*

By using similar methods (fractal theory and Matlab L-system simulation and holographic law) we get a leaf of ferns:

Graph 3: the leaf model produced via fractal theory

Further analysis:

We can classify leaves via their fractal dimensions either. Also, we can see the similarities between leaves and the tree from this creating method. Now comes to the calculation of mass of leaves in a tree.

Parameters:

Parameters	meanings
$\boldsymbol{m}$	Mass of one leaf
$\boldsymbol{N}$	The amount of all the generations apart from those serve as the leaves
$\boldsymbol{X}$	The generation number
$\boldsymbol{n_x}$	The generation number of each generation
$\boldsymbol{N_f}$	Number of symbol “f” in the iteration rule
$\boldsymbol{N_L}$	The amount of all the leaves in a tree
$\boldsymbol{M}$	Mass of all the leaves in a tree

According to the fractal theory, we have these formulae:

$$N_L = (N_f)^{(n+1)}$$

$$M = mN_L$$

Combine the two equations and we get an equation:

$$M = mN_f^{(n-1)}$$

Substituting the parameters with our data, take demonstration 1 for example, we have:

$$M = 5 \times 8^{6-1}g = 163840g = 163.840kg$$

As for demonstration2, by the same method, we have:

$$M = 5 \times 4^{6-1}g = 5120g = 5.120kg$$

Defects:

1. Some types of tree cannot fit well with the fractal model, such as willows and palm trees.
2. Actually, most trees are partially corresponding with our model. For instance, trees’ trunks are not that thin, so an improvement is a must so as to be suitable to the reality.

Leaf model:

Introduction of B-spline interpolation:

According to knowledge in mathematical analysis, we know that the ant derivative is more flexible than its corresponding function, which is to say, if:

$$\mathrm{f(x)} \in \mathrm{C}^{(\mathrm{k})}[\mathrm{a,b}]$$

Then easily we can get:

$$\int \mathrm{f(x)dx} \in \mathrm{C}^{(\mathrm{k+1})}[\mathrm{a,b}]$$

Thereby we can consider applying definite integral to improve the smooth degree of a function.

Definition: Assuming that f(x) is an integral function, for h>0, we call the integral:

$$f_{1,h}(x) = \frac{1}{h}\int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f(t)dt$$

One-generation polishing function, h is called the polishing width. Similarly, we call the integral

$$f_{k,h}(x) = \frac{1}{h}\int_{x-\frac{1}{2}}^{x+\frac{1}{2}} f_{k-1,h}(t)dt \ \ (k > 1)$$

as k-generation function.

Naturally, $f_{k,h}(x)$ has the higher polishing degree than f(x). Furthermore,

$$f_{k,h}(x) \approx f(x)$$

makes sense when h is little enough.

Isometric B spline function:

As for the function

$$\Omega_0 = (x + \frac{1}{2})_+^0 - (x - \frac{1}{2})_+^0$$

We know the graph is not complicated; it's a unit square-wave function. Making h=1, we make one polish on Ω_0 and we have $\Omega_1 = (x + 1)_+ - 2x_+ + (x - 1)_+$. Graph for this is a unit pointed-square.

We can polish Ω_0 till the NO.k generation and we have a function:

$$\Omega_k(x) = \sum_{j=0}^{k+1} (-1)^j \frac{C_{k+1}^j}{k!} (x + \frac{k+1}{2} - j)_+^k$$

The function has these properties:

$\Omega_k(x)$ is a segmentation of k-rank polynomial with (k-1)-rank continuous derivative.

The amount of k-rank derivative is (k+2).

$$x_j = j - \frac{k+1}{2}$$

and distance between each two derivative points is the same.

Therefore, we call it isometric k-rank B-spline function.

One-dimension isometric function interpolation:

Naturally, you may ask what association is between the B-spline and general interpolation function. Now we have a theory:

Theory: Assuming that $[a, b]$ is evenly devided: $\Delta: x_j = x_0 + jh \ (j = 0,1, \cdots, n), h = \frac{b-a}{n}$.

Then, any k-rank spline function $S_k(x) \in S_p(\Delta, k)$ can be expressed as the linear combination of B-spline functions

$$\left\{\Omega_k(\frac{x - x_0}{h} - j - \frac{k+1}{2})\right\}_{j=-k}^{j=n-1}$$

.

According to the theory, we can get

$$s_k(x) = \sum_{j=-k}^{n-1} c_j\, \Omega_k\left(\frac{x - x_0}{h} - j\right)$$

K=3 is a general situation and the formula is:

$$s_3(x) = \sum_{j=-k}^{n+1} c_j\, \Omega_3\left(\frac{x - x_0}{h} - j\right)$$

The amount of undetermined factors is (n+3).

Therefore, we have:

1. According to the interpolation conditions for the first questions of three-ranked spline interpolating function, we can get :

$$\begin{cases} s_3'(x_0) = \dfrac{1}{h}\displaystyle\sum_{j=-1}^{n+1} c_j\Omega_3'(-j) = y_0' \\ s_3(x_i) = \displaystyle\sum_{j=-1}^{n+1} c_j\Omega_3(i-j) = y_i \quad (i = 0,1,\cdots,n) \\ s_3'(x_n) = \dfrac{1}{h}\displaystyle\sum_{j=-1}^{n+1} c_j\Omega_3'(n-j) = y_n' \end{cases}$$

Tidying the equations, we have:

$$\begin{cases} -c_{-1} + c_1 = 2hy_0' \\ c_{i-1} + 4c_i + c_{i+1} = 6y_i \ (i = 0,1,2,\cdots,n) \\ -c_{n-1} + c_{n+1} = 2hy_n' \end{cases}$$

2. According to the interpolation conditions for the second questions of three-ranked spline interpolating function, we can get :

$$\begin{cases} c_{-1} - 2c_0 + c_1 = h^2y_0'' \\ c_{i-1} + 4c_i + c_{i+1} = 6y_i \quad (i = 0,1,2,\cdots,n) \\ c_{n-1} - 2c_n + c_{n+1} = h^2y_n'' \end{cases}$$

3. According to the interpolation conditions for the third questions of three-ranked spline interpolating function, we have :

$$\begin{cases} (c_{n-1} - c_{-1}) + 4(c_n - c_0) + (c_{n+1} - c_1) = 0 \\ (c_{n-1} - c_{-1}) + (c_{n+1} - c_1) = 0 \\ (c_{n-1} - c_{-1}) - 2(c_n - c_0) + (c_{n+1} - c_1) = 0 \\ c_{i-1} + 4c_i + c_{i+1} = 6y_i \ (i = 0,1,2,\cdots,n) \end{cases}$$

Generally speaking, B-spline is more steady, where 3-ranked calculation and fitting are both appropriate and the 4-ranked calculation is kind of complicated but has higher fitting accuracy.
This method is more concise as well as intuitive than other tools such as Matlab.

Assumptions:

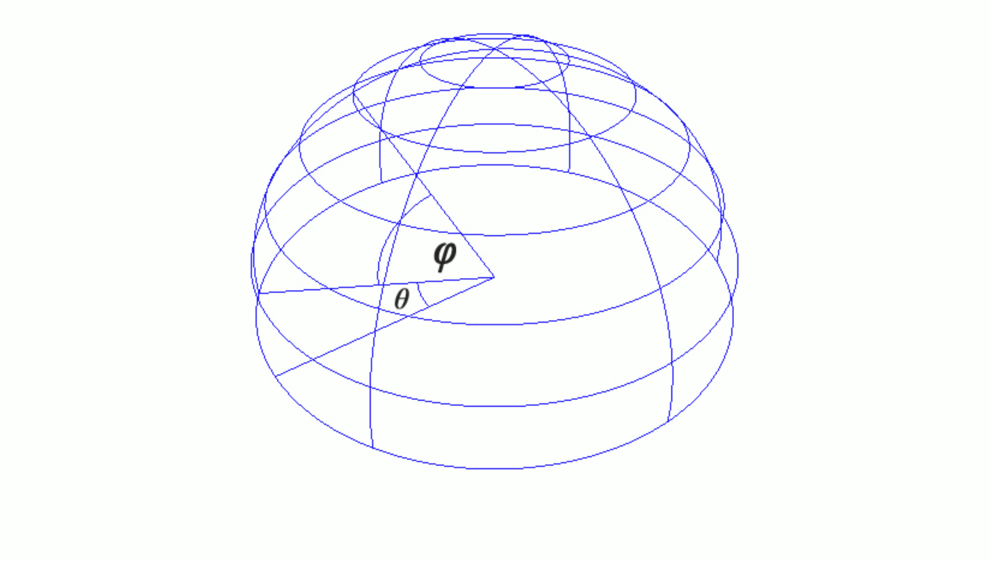
1. Our tree model is developed basing on the fractal model.
2. The tree looks indiscriminately in every angle.
3. Leaf shape is elliptic.
4. The organism fits with principle of Maximum illumination area.

Parameters:

Parameters	meanings
$\boldsymbol{\theta}$	Horizontal plane angle
$\boldsymbol{\varphi}$	Vertical plane angle
$\boldsymbol{S_T}$	Projection acreage of the tree(varies with the illumination angle)
$\boldsymbol{S_{TMAX}}$	Maximum of the tree's projection acreage
$\boldsymbol{A}$	Long axis of one leaf
$\boldsymbol{B}$	Short axis of one leaf
$\boldsymbol{E}$	$\boldsymbol{e} = \frac{\boldsymbol{a}}{\boldsymbol{b}}$ (according to biological discipline, the minimum of e equals one)
$\boldsymbol{S_L}$	Projection acreage of the tree varies with e.
$\boldsymbol{S_{LMAX}}$	Maximum acreage of the tree when the illumination angle stay unchanged (corresponding to a certain value for "e")
$\boldsymbol{D}$	Distribution of the leaves on the tree

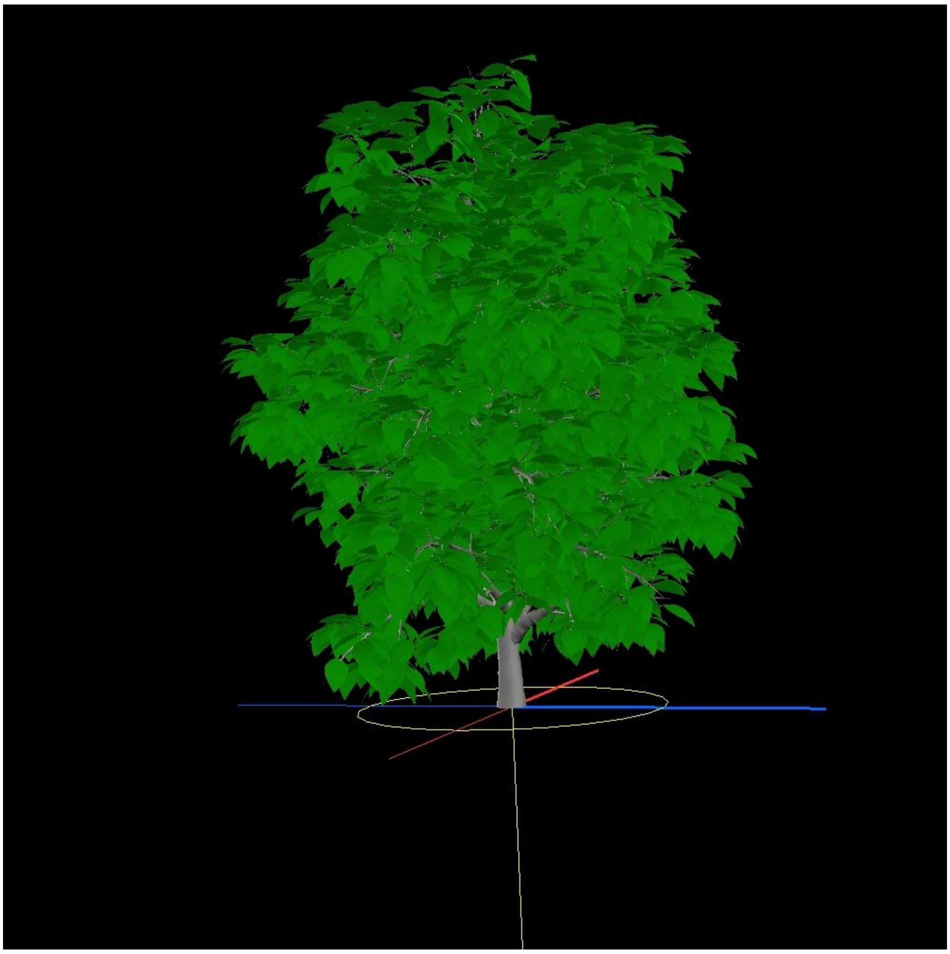
Part one: Steps for the setup of our model:

Firstly, we draw an all-perspective graph via Grapher as graph 4 shows.



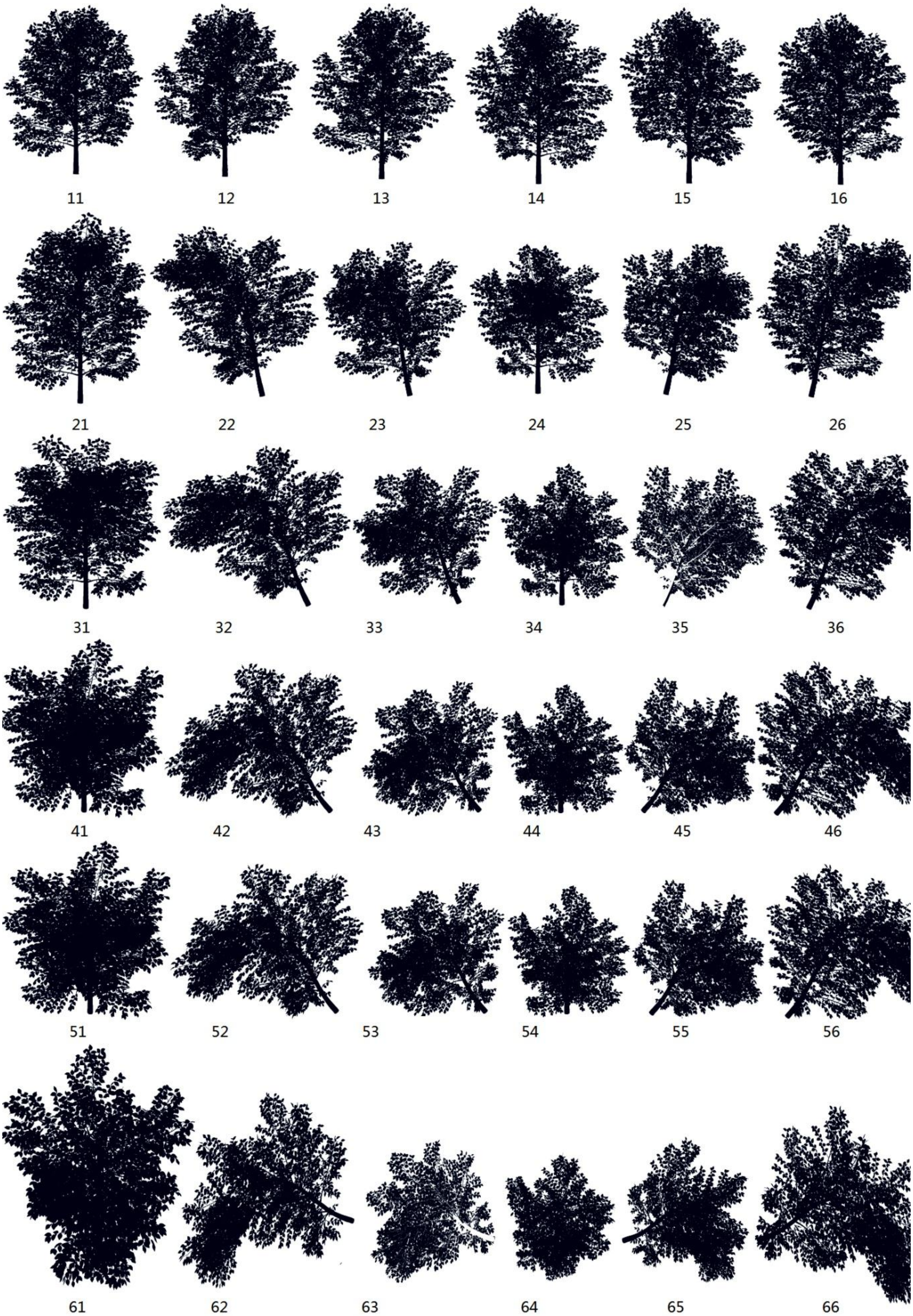
Graph 4: All-perspective graph

Secondly, we simulate a chestnut tree via XFROG.

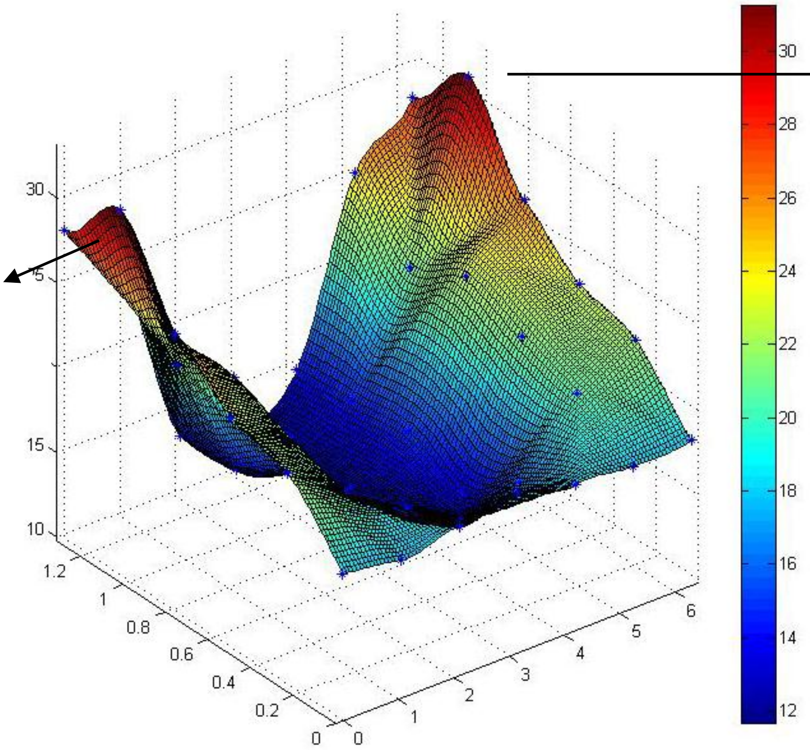


Graph 5: Simulation of chestnut tree basing on fractal theory

Thirdly, with the help of PS, Matlab and B-spline interpolation, we adjust the illumination angle and choose an angle corresponding with the maximum of projection area finally. Here is our result in Matlab.



Graph 6: shadows of the tree under different illumination angles



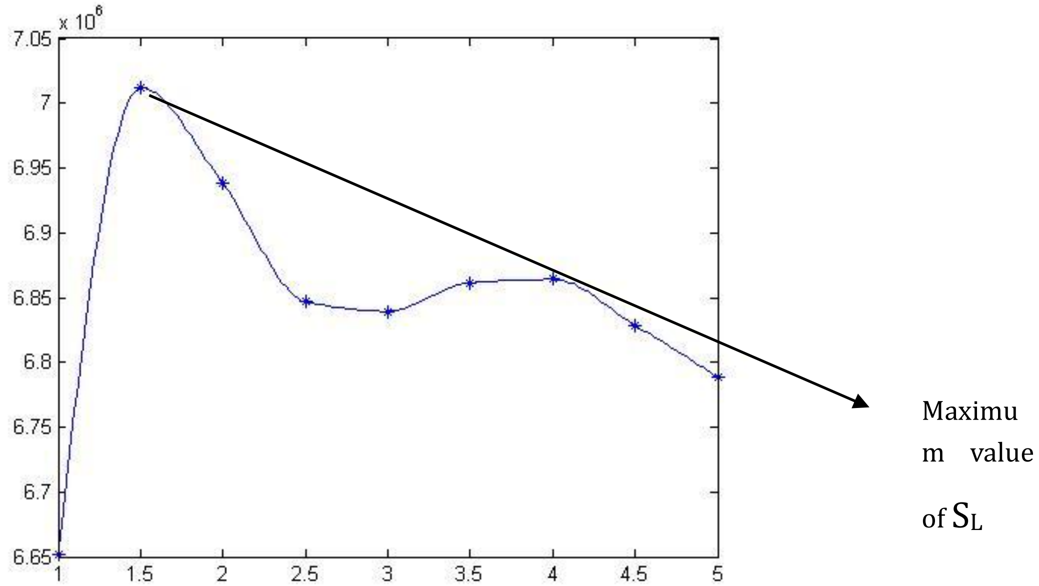
Graph 7: 3D functional graph: mapped from (θ,φ) *to* S_T

Here is our data table:

θ	φ	S_T	θ	φ	S_T	θ	φ	S_T
0	0	18.4022	0	0.5236	23.2764	0	1.0472	31.2482
1.0472	0	17.9072	1.0472	0.5236	20.8142	1.0472	1.0472	20.8415
2.0944	0	18.5131	2.0944	0.5236	15.6682	2.0944	1.0472	13.3587
3.1416	0	19.2350	3.1416	0.5236	13.9873	3.1416	1.0472	12.4794
4.1888	0	18.3955	4.1888	0.5236	13.6970	4.1888	1.0472	14.8021
5.2360	0	18.1388	5.2360	0.5236	21.4996	5.2360	1.0472	21.2746
6.2832	0	18.4022	6.2832	0.5236	23.2764	6.2832	1.0472	31.2482
0	0.2618	22.1631	0	0.7854	26.0926	0	1.3090	27.8970
1.0472	0.2618	19.6734	1.0472	0.7854	22.2427	1.0472	1.3090	23.8525
2.0944	0.2618	16.6176	2.0944	0.7854	15.3774	2.0944	1.3090	13.1540
3.1416	0.2618	15.4887	3.1416	0.7854	12.9792	3.1416	1.3090	11.9069
4.1888	0.2618	16.4189	4.1888	0.7854	15.1431	4.1888	1.3090	14.4883
5.2360	0.2618	20.2940	5.2360	0.7854	22.9041	5.2360	1.3090	24.7669
6.2832	0.2618	22.1631	6.2832	0.7854	26.0926	6.2832	1.3090	27.8970

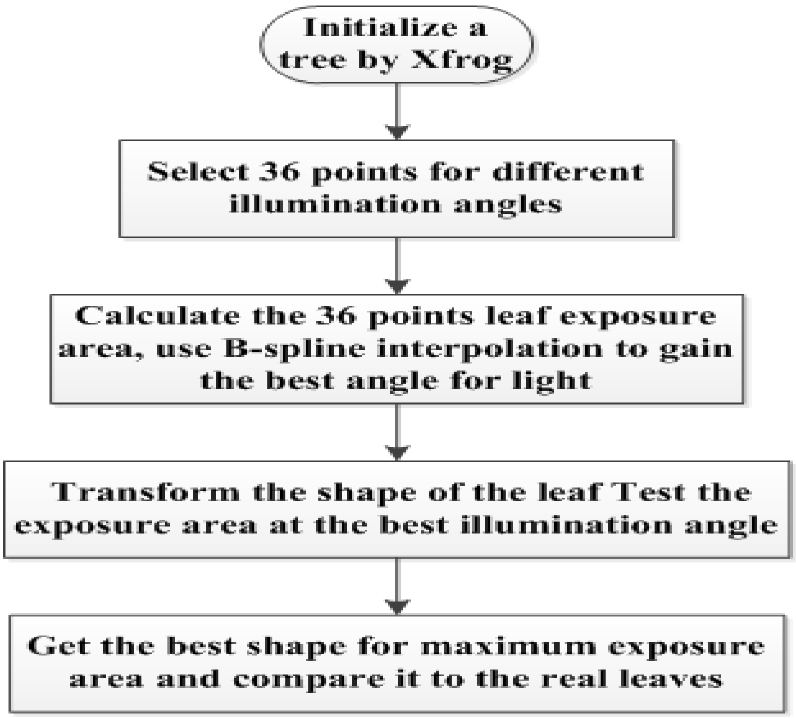
After fitting and integration, we know that while $\varphi = \frac{\pi}{3}$ and $\varphi = 0$ the acreage reaches the maximum. It is worth mentioning that $\varphi = \frac{\pi}{3}$ corresponds with our daily experience and biological theories which says photosynthesis rate reaches maximum at 2 or 3 pm instead of at midday or other time in a day corresponding with angles apart from $\frac{\pi}{3}$.

Fixing the illumination angle, we change the proportion of long and short axis of the leaf whose acreage stay unchanged and then we gain a figure as showed below.



Graph 8: $e - S_L$ Functional graph

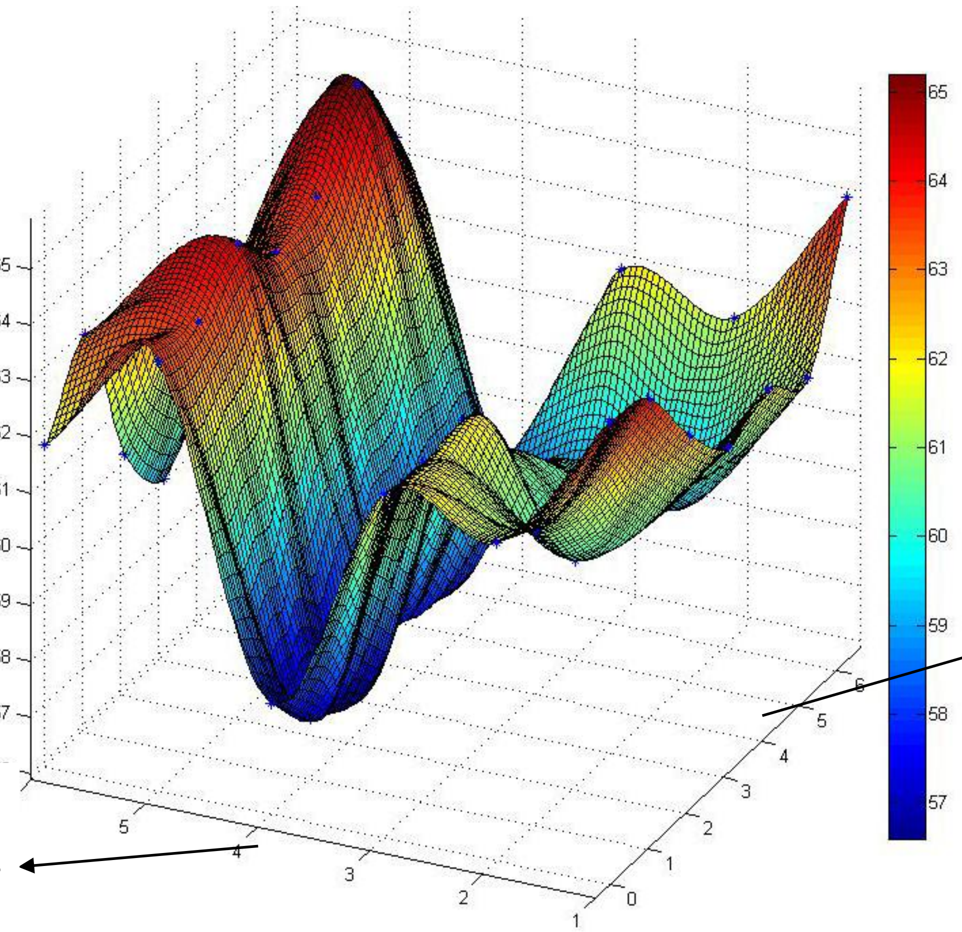
We take 1.5 as the best value for e and the shape is fixed till now.
In order to clearly explain our work, we work out a flow chart:



Graph 9: Flow chart for the leaf model-part one

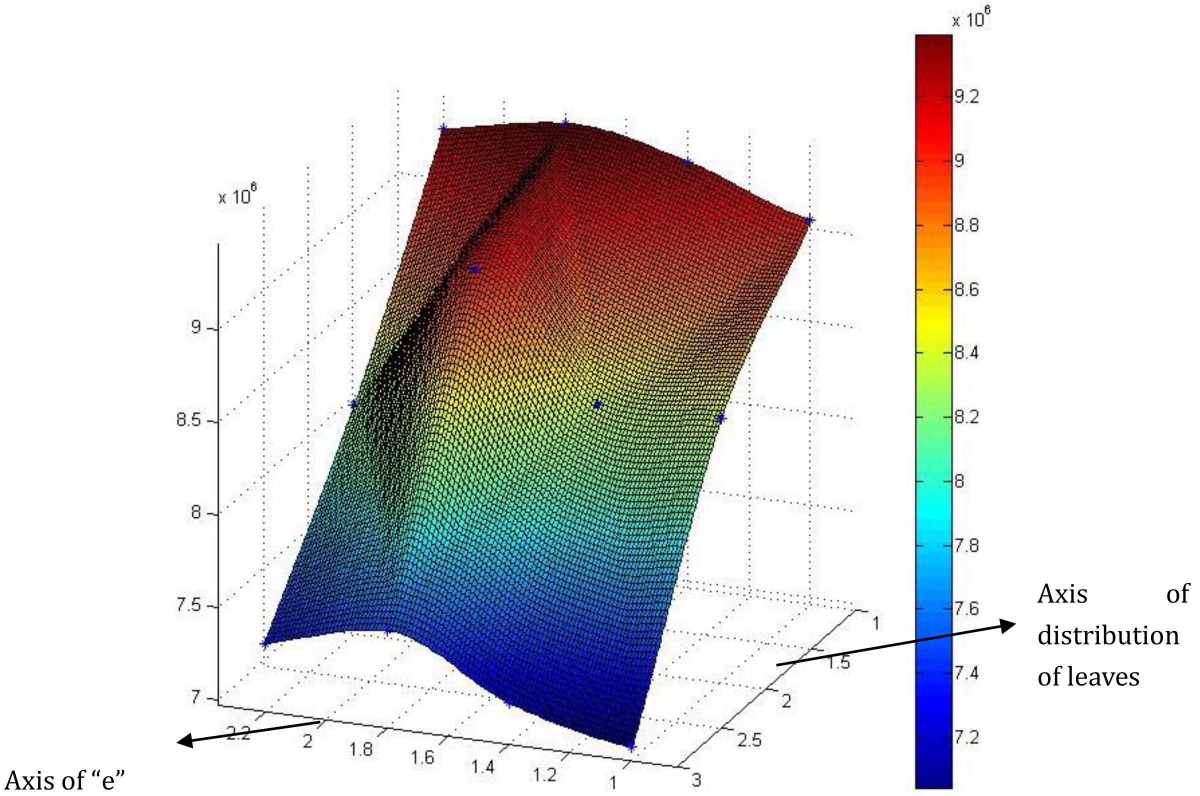
Part two:

After changing the distribution of the leaves on the tree, we obtain these two graphs accounting for leaf shape's successive change with the change of leaves' distribution.



Graph 10: $D - S_T$ *functional graph*

This shows that S_{LMAX} varies with distribution of leaves.



Graph 11: D-e functional graph

The graph tells us leaves' shape varies obviously with the alternation of distribution of leaves.

Relation between leaf and tree's profile:

Holographic law was first discovered by Zhang Yingqing in his research in 1973 by the method of association and analogy. Afterwards the law was spread to the field of botany. It can be expressed as follows: system that holds obvious boundaries in function and structure towards its surrounding parts, we call it a relatively independent part. Each of these systems is highly similar to the whole organism on most aspects.

We think leaf shape related to tree profile. Here is the proof. Firstly, we all know that a leaf is in charge of transporting water and nutrient. In the meantime, branches hold the responsibility of transporting water and nutrients as well. Structure determines function; thereby the leaf and the tree ought to have similar profile characteristics. Secondly, according to Holographic law, a leaf is a relatively independent part apparently and we acknowledge the high similarities between a leaf and its corresponding tree.

Thirdly, based on the previous work, we produced a new model.

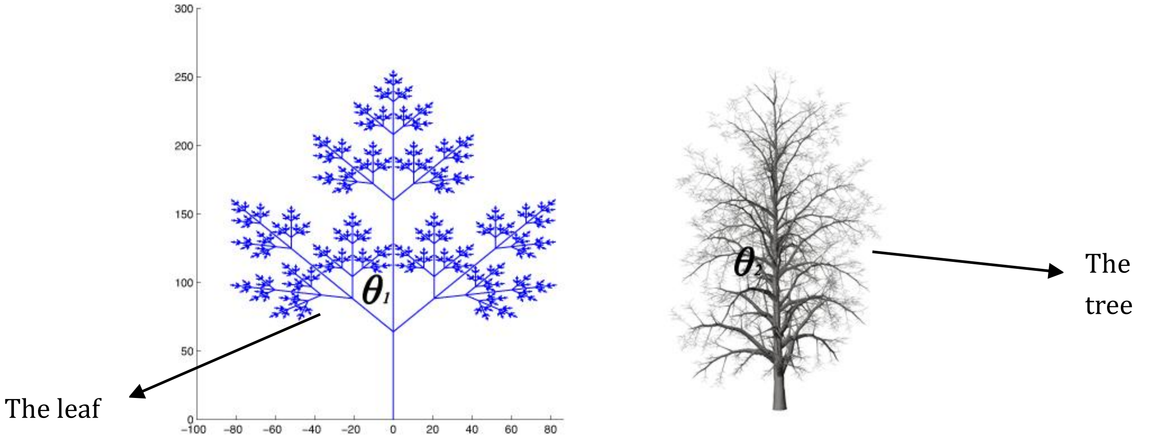
Our model

Assumptions:

The tree grows in a natural environment without manual interfere.

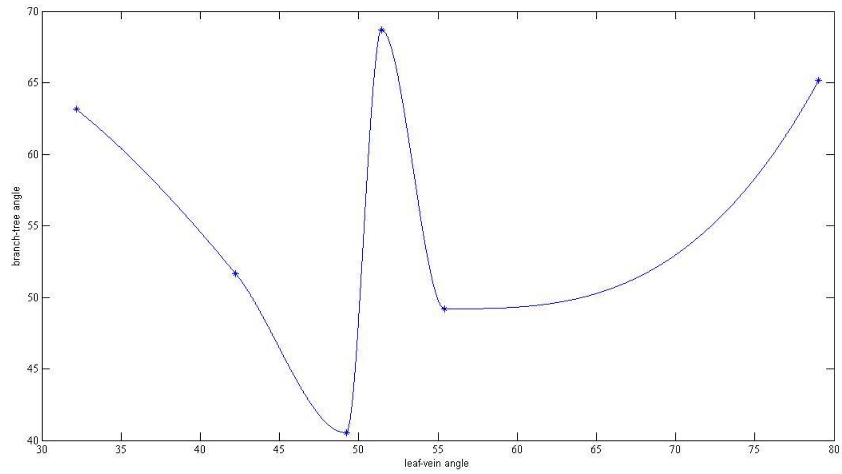
Parameters:

parameter	meanings
$\boldsymbol{\theta_1}$	Deflection angle of the leaf
$\boldsymbol{\theta_2}$	Deflection angle of the tree



Graph 12: contrast graph between a model tree holding the leaf's deflection angle (left) and its corresponding model tree created by Xfrog (right)

From the graph above we can see a leaf is related to its corresponding tree to some degree. Now comes to the specific steps of our model. First, we collect data. Second, with the help of fitting and interpolation, we get a graph showing relation between $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$.



Graph 13: $\theta_1 - \theta_2$ *functional graph*

After plenty of data collection and analyses, we find that the discrepancy between $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ is quite little as graph 11 shows.

Correlation between the leaf mass and the tree profile:

We consider the problem standing on a revised model basing on the tree model referred before.

Assumptions:

1. Trees fit well with our model.
2. Real tree height corresponds with Logistic model.
3. The Logistic model expounds association between tree's age and its height in reality quite well.
4. The tree grows slower and slower as time goes by. （fits the index model）

Parameters:

parameters	meanings
$\mathbf{H_T}$	Relative height of the tree in our tree model
A	Age of a tree
α	We call it "deflection angle" (it's showed in Graph 11)
H	Real height of a tree (height in Logistic model)
w	The ratio of increment height to that of the previous generation
$\mathbf{H_0}$	Height of the first generation of the tree model
K	Temporary unknown parameter
a	Temporary unknown parameter
b	Temporary unknown parameter
$\mathbf{H_{Fn}}$	Tree height of the nth generation
β	The ratio between tree's age and generation number in our fractal tree model

According to L-system theory, we have:

$$H_{Fn} = wH_{F(n-1)}cos\alpha$$

Then we can get:

$$H_F = H_0 + wH_0cos\alpha + \cdots + w^{n-1}H_0cos(n\alpha)$$

That is,

$$H_F = H_0(1 + wcos\alpha + \cdots + w^{n-1}cos(n\alpha))$$

for which a more concise form is as below:

$$H_F = H_0(cos\alpha \quad \cdots \quad cos(n\alpha))\begin{pmatrix} 1 \\ w \\ \vdots \\ w^{n-1} \end{pmatrix} \cdots (1)$$

What's more, we know that tree height corresponds with this model:

$$H = \frac{K}{1 + ae^{-bA}} \cdots (2)(Deng\ Hongbing\ etc. 1999)$$

Model one for question 3:

According to our tree model, we have:

$$M = mN_L \ \ and \ \ N_L = N_f^{n-1}$$

Then we have：

$$M = mN_f^{n-1}$$

We change the independent variable and the dependent variable, and then we have:

$$n = log_{N_f}\left(\frac{M}{m}\right) + 1 \cdots (3)$$

Substituting the (3) equation into equation (1), we have:

$$H_F = H_0\left(cos\alpha \quad \cdots \quad cos\left(\left(log_{N_f}\left(\frac{M}{m}\right)+1\right)\alpha\right)\right)\begin{pmatrix} 1 \\ w \\ \vdots \\ w^{log_{N_f}\left(\frac{M}{m}\right)} \end{pmatrix} \cdots (4)$$

This is the correlation between M and HF.

Model two for question 3:

$$A = \beta e^{-n}$$

According to our assumption, we have: H=HF. Then we have the equations below:

$$\begin{cases} H = \dfrac{K}{1 + ae^{-bA}} \\ M = mN_f^{n-1} \\ A = \beta e^{-n} \end{cases}$$

Consider them together, we can get:

$$H = \frac{K}{1 + ae^{-b\beta e^{-\left[log_{N_f}\left(\frac{M}{m}\right)+1\right]}}}$$

This shows the correlation between M and H.

It is not negligible that parameters K, a and b are const in our model since Deng Hongbing has got their values by the method of regression analysis method in his research. (K=26.958, a=8.96, b=0.025 for pinus koraiensis)

Summary

Why is the leaf the way it is? Why do leaf has its shape and structure? We set models to solve this problem. Our models are under general assumption below:

1. Tree model match the fractal conditions; self-similar, and infinite grow;
2. Leaf has two basic characteristic: vein structure and shape;
3. Leaf can be regarded as ellipse when considered about the relationship between shape and illumination.
4. Tree growth obeys the Logistic equation.

When consider about the illumination angle for the exposure angle, we simulate real tree on the platform of Xfrog. This software offers relatively reality and 3D view of the tree. And we use interpolation fitting to study the complex relationship between maximum exposure area and illumination angle.

Our results are simple but direct. First, we proved that the leaf shape does ensure maximum exposure and we even get the best sunlight angle in the ideal environment. Further, we discover that the distribution of leaves among the tree height affects the leave shape a lot. Secondly, we study the relationship between leaf shape and tree profile and branch structure, finding an imperfect interpolation result. Next, we develop a method to calculate the leaf mass by counting the leaf quantity. Then, we combine our models and others to acquire the relationship between leaf mass and the tree size characteristics. And in all, we proposed two factors for leaf classification: leaf shape and vein's structure, which are connected to other parts of the tree.

We mostly apply our models to the simulation of the tree generated by Xfrog. It is a virtual simulation, so we can only get the "experimental" value instead of the real value. However, this software is of enough accuracy. Thus, we believe the error range is tolerable.

Strength:

1. The fractal model is popular lately and is believed to obey the real nature of plants.
2. Our way to describe leaf is vivid and easy to understand.
3. Our simulation by Xfrog is very vivid and corresponds to our model well.

Weakness:

1. The leaf mass data are not available so that we cannot test the error data of leaf mass calculation.
2. The model of fractal tree is two-dimensional, simplifying the real situation, causing error data.
3. The leaf vein's structure only consider about the angles, not about other factors.

Using fractal to study plants problems has been a trend. Many people do research about the relationship between fractal dimensions and plants characteristics. We raise a different model to consider these factors.

Letter:

Dear editor:

I am writing to you to introduce to you our key findings about the research on leaf shape and tree profile. Here are our key findings.

We make two basic models and then several models basing on the basic two. One of our basic models is called “tree model” and the other “leaf model”. Tree model is a production of fractal theory and L-system and we simulate a tree’s growth process by Matlab while leaf model is the production of fitting and interpolation. Tree model can be used to calculate mass of leaves and leaf model can be used to classify as well as describe leaves. We work out the leaves’ amount via the assumptions. Finally, we calculate out leaves’ mass via a basic formula.

Our leaf model is somewhat more elaborate. This model can be used to classify leaves. Firstly, we draw an all-perspective three-dimensional figure. Secondly, we simulate a chestnut tree via Xfrog. Thirdly, with the help of PS, Matlab and B-spline interpolation method, we adjust the illumination angle and choose an angle corresponding with the maximum of projection area finally. After these “artful” steps, the best leaf shape is fixed whose ratio of long axis to short axis is 1.5. It is gratifying that the result fits well with the reality. Then we adopt our previous and acknowledge distribution of leaves’ effecting on the shape of leaf.

Afterwards, we produced several models on the bases of the two former models. The first model tells us there be relation between leaf shape and branching structure. What’s more, the deflection angle can be seen as one classify criterion for classifying the leaves. As for the correlation between leaf mass and the size characteristics of the tree, we make further revisions to our tree model and we combine our new model with a Logistic model for the height of the tree. At last, we gain two models acknowledging correlation between the leaf mass and the size characteristics of the tree.

We also insert certain amount of vivid simulation graphs in the paper and they make our model more clear.

Best wishes!

2012/2/14

Team: 15177

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