Report on recent activities

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symbolic methods in kinematics

From May of the year 2018, I started working on the project "How to avoid collision for the 3D-realization of a moving graph", which can be summed up as follows:

- Parameterizing the positions of vertices of a graph gives us a moving graph.
- Assigning different heights to the edges of a moving graph gives us an L-model.
- The 3D-realization of an L-model is called an L-linkage.
- A property of the moving graph guarantees the collision-freeness of its L-linkage.
- An algorithm for building this collision-free linkage is provided.

The result is published in *Mechanism and Machine Theory 162 (2021):104337*.
`https://doi.org/10.1016/j.mechmachtheory.2021.104337`

moduli space

From January of the year 2019, I started working on the project "moduli of n points on the projective line" with Prof. Herwig Hauser and Prof. Josef Schicho. The project is still ongoing, the content of which can be summed up as follows:

- Let $n \geq 3$ be an integer, we study the equivalence induced by the group action of $PGL(2, \mathbb{C})$ on $(\mathbb{P}^1)^n$.
- Two n-tuples are equivalent if there is a projective linear transformation transforming one into the other.
- In our setting this transformation is nothing more than Möbius transformation.
- When the n-tuples have n distinct points, the equivalence classes are in bijective correspondence with the points of an open subset $(\mathbb{P}^1)^{n-3}$, which can be parametrized by $n-3$ cross ratios.

moduli space

- We denote by U_n this open set, which is defined as the open subset of all points $(c_4, \dots, c_n) \in (\mathbb{P}^1)^{n-3}$ where $c_i \notin \{\infty, \mathbf{0}, \mathbf{1}\}$ for $i \in \{4, \dots, n\}$ and $c_i \neq c_j$ if $i \neq j$, where $i, j \in \{4, \dots, n\}$.
- U_n is the moduli space of n distinct points on $\mathbb{P}^1$, under PGL_2 group action.
- It is an open subset of $(\mathbb{P}^1)^{n-3}$, and $(\mathbb{P}^1)^{n-3}$ is indeed a compactification of it, which is projective and smooth. However, the first three entries are somehow special, so it is not symmetric under random permutation of the nodes.
- We want to find a good compactification of U_n which is smooth, symmetric under permutation of nodes, projective.

Prof. Hauser is giving a course on this project, for more details see `https://www.xxyyzz.cc/`.

intersection theory of moduli spaces

- In spring/summer of the year 2019, I was trying to improve an algorithm by Matteo, Georg, al. on counting the realizations of a Laman graph on the sphere.
- I did not manage to really improve it, but I found out another algorithm which can do the same thing.
- As a side-product, a neat algorithm (which serves as a sub-step for the Laman graph realization counting) on computing the integral value of monomials in Chow group of zero cycles in the moduli space of stable curves of genus zero surfaced.
- With the help from Josef on the correctness proof, the project was completed.

intersection theory of moduli spaces

The content of this project can be summed up as follows:

- The Chow group of zero cycles in the moduli space of stable pointed curves of genus zero is isomorphic to the integer additive group.
- Let M be monomial in this Chow group. If no two factors of M fulfill a particular quadratic relation, then the monomial can be represented equivalently by a specific tree; otherwise, M is mapped to zero under the stated isomorphism.
- Starting from this tree representation, we introduce a graphical algorithm for computing the corresponding integer for M under the aforementioned isomorphism.
- The algorithm is linear with respect to the size of the tree.

As a poster accepted by ISSAC2020, the extended abstract was published in *ACM Communications in Computer Algebra 54, no.3(2021): 91-94*.

side products

- During the study on the above project, an identity on multinomial coefficients was discovered.
- Together with Dongsheng Wu (Tsinghua University), we proved it.
- The identity and the proof can be found in Section 7 of the preprint (arXiv:2101.03789).
- During the study of above two projects, we found five equivalent representations of a phylogenetic tree.
- Together with Josef, we finished the investigation on these representations, formalized the results in the preprint (arXiv:2011.11774).

graph theory

In the AEC summer school 2019, I met Jovana Forcan and got to know her work about maker–breaker domination number. I was very much interested in it, and we started working together on some open problems in the field. Very nice results were found, and they are formalized in the preprint (arXiv:2004.13126). The content can be summed up as follows:

- A game is played on a given graph by two players who take turns to claim a vertex that has not yet been claimed during the game.
- One is called Dominator and another Staller.
- Dominator wins if he can claim a domination set at some point, Staller wins if she can prevent Dominator from winning.

graph theory

- The minimum number of moves for Dominator to win the game on a given graph is an invariant for the graph.
- This number is denoted by $\gamma_{MB}(G)$ when Dominator is the first one to play, and by $\gamma'_{MB}(G)$ when Dominator is the second to play.
- Note that this number is usually denoted as ∞ if Dominator does not have any winning strategies for the game.
- This number is finite when he has a winning strategy.
- We figured out the domination number for $P_2 \Box P_n$.

moduli spaces and phylogenetic trees

- The current title of the thesis is "moduli spaces and phylogenetic trees".
- The plan is to put in the results of two projects among the mentioned ones in the thesis.
- Namely, "the integral value of monomials in Chow group of zero cycles in the moduli space of stable curves of genus zero", and "five representations for a phylogenetic tree".

some new problems?

- Currently, I am working on the maker–breaker domination number for $P_3 \square P_n$.
- Besides, I intend to investigate more into the intersection theory of moduli spaces of stable curves of non-zero genus.

Thank You